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Experimental Study of Wind-Wave Interaction

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Introduction

THIS brief Note is written to report some of the experimental results of the study on the generation of water surface waves by wind. Various theoretical and experimental investigations have been performed in the past.[1–11] Interest in the problem dates from the work of Helmholtz[1] and Kelvin[2] whose mathematical modelling of the flow yielded a prediction for the critical wind velocity, or as it is called, the onset of instability, required to generate surface waves. For air over water this prediction yields a value of 21.3 fps. On the open ocean or lake, observations show that the critical velocity is rather nearer 0.68 fps. In a recent paper,[3] in which the air is modelled as stratified, two critical velocities are revealed; one at 0.705 fps (the "initial instability") and the second at 19.9 fps (the "gross instability"). Laboratory observations have ranged from 0.722 fps to as high as 21.35 fps for different depths of water. This Note reviews these measurements as well as reports on an investigation of the influence of corrugated bottom geometry on the critical wind velocity.

Experimental Apparatus

A wind tunnel fitted with a water tank, shown in Fig. 1, was designed and built to study the critical velocities for instabilities at the wind-water interface. A "Flow Corporation" Model WT4 wind tunnel was used as an exhaust blower to produce air flow over the water. It is fitted with a base and supported by four helical springs to minimize vibration. The blower is equipped with a variable speed controller.

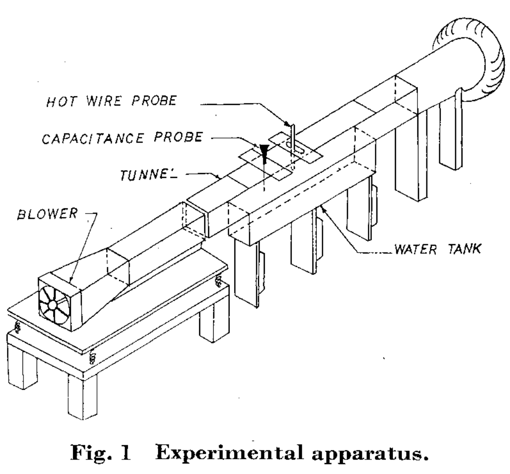

Fig. 1 Experimental apparatus.

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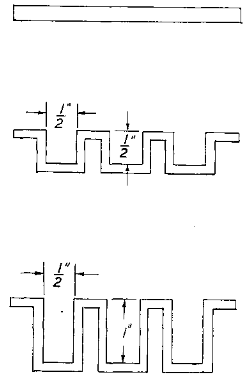

Fig. 2 Configurations of different shapes of beds.

The water tank section was fabricated from Plexiglas and it is 3.16 × 3.16 × 30 in. on top for air passage and fitted underneath with a 3.16 × 4 × 18 in. water tank. A variable bottom is supported by three screws so that the water depth may be varied. Tap water is introduced into the bottom of the water tank for fill and through a needle valve to the top of the water tank to compensate for evaporation.

Instrumentation and Experimental Procedures

The water tank is filled with tap water until the water level coincides with the inside bottom surface of the air passage. After the water level has settled, a capacitance probe is immersed 0.125 in. beneath the water surface. Flow of the airstream is then initiated in the tunnel and equilibrium conditions are allowed to be reached. The output signal of the capacitance probe due to change of the level of the water surface is sent into an oscilloscope and is recorded photographically. This procedure is performed over a range of air flow speeds for various water depths and three different shapes of beds (see Fig. 2). In every experiment these streamwise corrugations were totally immersed. For determining the velocity and the turbulent intensity level in the airstream corresponding to various settings of the flow controller, a constant temperature hot-wire anemometer system with a linearized anemometer unit is used. These measurements are made at various positions in the airstream above the water surface. In addition, the frequency spectrum of the air turbulence is obtained by recording the turbulent signal using a frequency modulated tape recorder, and then analyzing it with a wave analyzer. The turbulence spectrum is then recorded and reduced to energy per cycle as shown in Fig. 3. Because of the

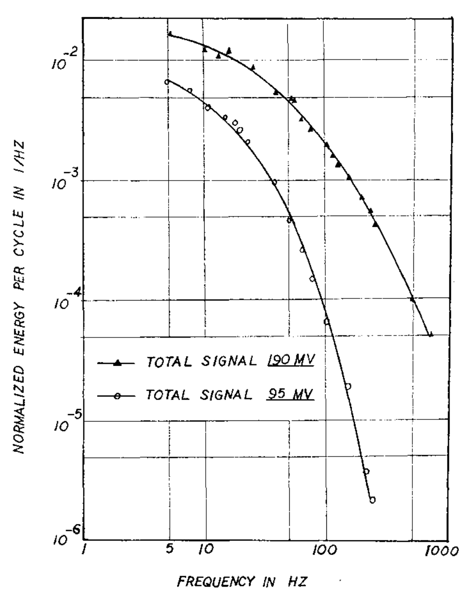

Fig. 3 Energy-frequency spectrum of turbulence in air stream.

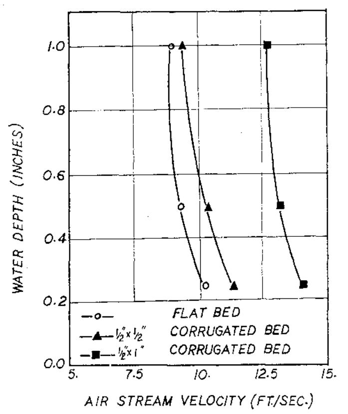

Fig. 4 Dependence of second instability on water depth and bed configurations.

limitation of the frequency response of the wave analyzer, however, only signals down to 5 cps can be detected.

Results and Discussion

The results of the present investigation reveal that there exist two different modes of instabilities of the wind-water interface. The first one occurs at an air velocity of approximately 2.42 fps. The surface waves due to this first instability are of extremely small amplitudes and can be observed visually only by very careful examinations. Furthermore the onset of this instability is found to be independent of the water depth and/or the configuration of the bed. The displayed patterns of the surface waves of the water remain fairly constant after the occurrence of the first critical velocity, except the gradual growth of their amplitudes with increasing speed of the airstream. The surface waves become erratic, however, when the second instability, or "gross instability" as it is called, is reached. The airstream velocities corresponding to this second instability are found to vary from 9.16 to 13.9 fps for different depths of water and different shapes of the bed configuration. This dependence of the second instability on water depth and bed configuration is shown in Fig. 4. The frequency of the water wave at the second instability is found to be approximately 5 cps which corresponds to the highest energy component in the spectrum of the turbulent airstream as shown in Fig. 3.

Conclusion

In conclusion, two different types of instabilities are observed in the present study of wind-wave interaction corresponding approximately to the prediction made by Sontowski[3] et al., Miles,[4-6] and Liang[7] et al. The first instability, occurring at an airstream velocity of 2.42 fps, is found to be independent of the depth of the water and the shape of the bed. The second instability, which gives an erratic wave motion of the water surface, is found to occur at air velocities of 9.16 to 13.9 fps depending on the depth of the water and the shape of the bed. The surface wave at this instability is observed to have a frequency of approximately 5 cps which corresponds to the strongest frequency component in the air turbulence spectrum (see Fig. 3). It suggests clearly, therefore, a one-to-one frequency correspondence for the exchange of energy between the airstream and the water wave at the onset of the second gross instability.

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Further Results on Recurrent Lagrange Multipliers for the Low-Thrust Earth-Jupiter Transfer

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IN Ref. 1, it was shown that some of the initial Lagrange multipliers along with the mission durations for constant power low-thrust Earth-Jupiter transfers are recurrent functions of the launch date. The multipliers which exhibited recurrent behavior were those associated with the motion of the transfer vehicle parallel to the ecliptic plane. The model studied employed inertial rectangular cartesian coordinates to

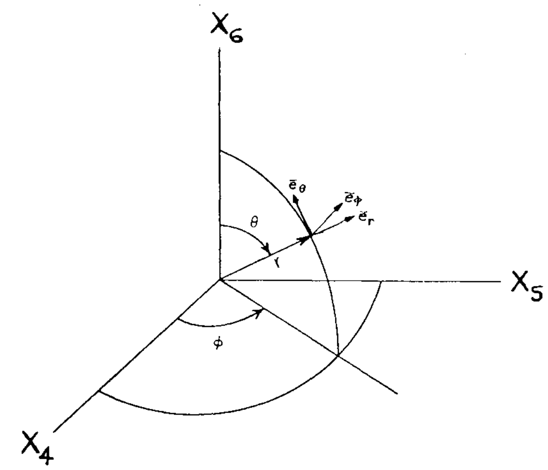

Fig. 1 Spherical coordinate system.

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